Math 10A HW1 Solutions

(1) True

False

True $(2 \notin B$ so $2 \notin A \cap B)$

True $(7 \notin A, 7 \notin B)$

(2) False (animals aren't plants!)

False (every bird IS an animal)

False (birds are animals but not mammals)

True (birds & mammals are both animals)

(3) (a) Yes function

Domain: $\{1, 5, 6, 7\}$

Range: $\{2, 3, 6, 9\}$

(b) No, use VLT

(c) No, use VLT

(d) Yes function

Domain: $\mathbb{R}$

Range: $[2, \infty)$

(e) Yes function

Domain: $\mathbb{R}$

Range: 5

(4) False! VLT is used to check if curve on plane is a function.

(5) (a) $(g \circ f)(3) = g(3^2+1)$

$= g(10)$

$= 10-2$

$= 8$

(b) $(g-f)(3) = g(3) - f(3)$

$= (3-2) - (3^2+1)$

$= 1 - 10$

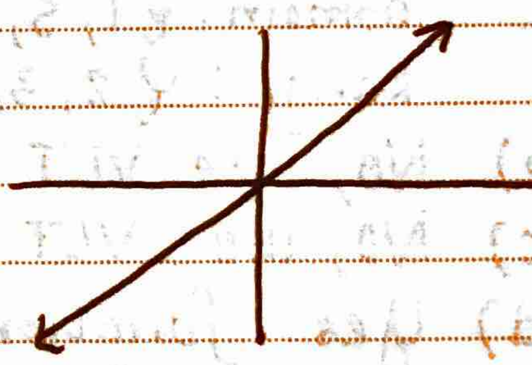
$= -9$

(c) $(g/f)(3) = g(3)/f(3)$

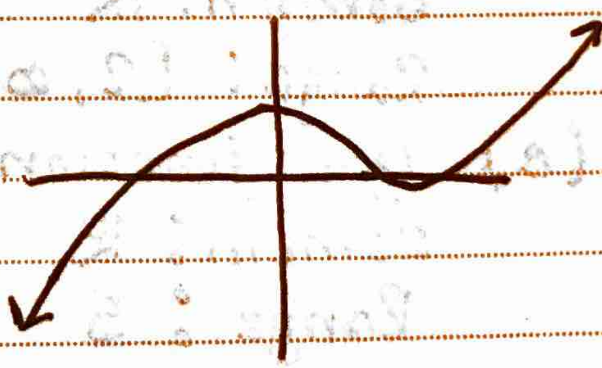
$= (3-2)/(3^2+1)$

$= 1/10$

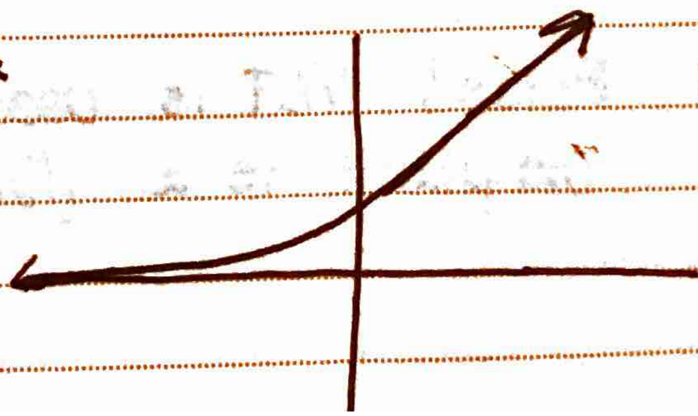
(6) (a) Let $f(x) = x$.



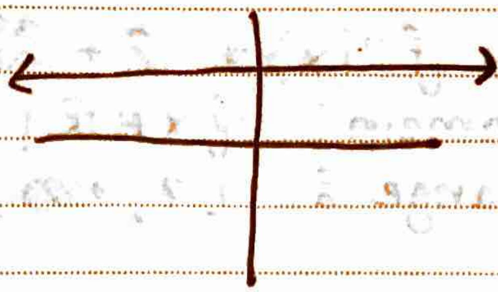
(b)



(c) Let $f(x) = e^x$



(d) $f(x) = 1$



(7) (a) $y = 3x+1 \longleftrightarrow x = 3y+1$ (switch variables)

$y = \frac{x-1}{3}$ (isolate for y)

So $f^{-1}(x) = \frac{x-1}{3}$.

Domain: $\mathbb{R}$
Range: $\mathbb{R}$

(b) $y = e^{2x+1} \longleftrightarrow x = e^{2y+1}$

$\ln(x) = \ln(e^{2y+1})$

$\ln(x) = 2y+1$

$y = \frac{\ln(x)-1}{2}$

So $f^{-1}(x) = \frac{\ln(x)-1}{2}$.

Domain: $\{x \in \mathbb{R} \mid x > 0\}$
Range: $\mathbb{R}$

(c) $y = (x-2)^3+1 \longleftrightarrow x = (y-2)^3+1$

$x-1 = (y-2)^3$

$\sqrt[3]{x-1} = y-2$

$y = 2 + \sqrt[3]{x-1}$

So $f^{-1}(x) = 2 + \sqrt[3]{x-1}$.
Domain: $\{x \in \mathbb{R} \mid x \geq 1\}$
Range: $[2, +\infty)$

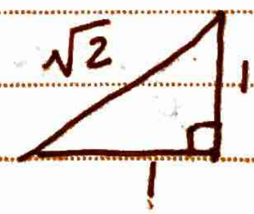
(d) $y = x/x+1 \longleftrightarrow x = y/y+1$

$$xy + x = y$$
$$x = y - xy$$
$$x = y(1-x)$$
$$y = x/1-x$$

So $f^{-1}(x) = x/1-x$
Domain: $\{x \in \mathbb{R} \mid x \neq 1\}$
Range: $\{\cancel{x}y \in \mathbb{R} \mid y \neq -1\}$

(8) Notice $f(7.5) = 500$. Thus $f^{-1}(500) = 7.5$.

(9) (a) $9 \cdot 3^5 = 9 \cdot 243 = 2187$
(b) $(2^1)^2 = 2^2 = 4$
(c) $2^{(1^2)} = 2^1 = 2$
(d) $\ln(1) = 0$
(e) $\ln(e) = 1$
(f) $\ln(2) + \ln(1/2) = \ln(2 \cdot \frac{1}{2}) = \ln(1) = 0$
(g) $\cos 0 = 1$
(h) $\sin(\pi/4) = 1/\sqrt{2}$



(i) $\cos(\pi/2) = 0$

(j) $\sin(-\pi/2) = -1$

(10) True; by definition

(11) False; let f be the constant function s.t. $f(x) = 20$. Then f is not invertible however there is a solution to $f(x) = 20$ (namely any $x \in \mathbb{R}$).

(12) True; by definition of inverse

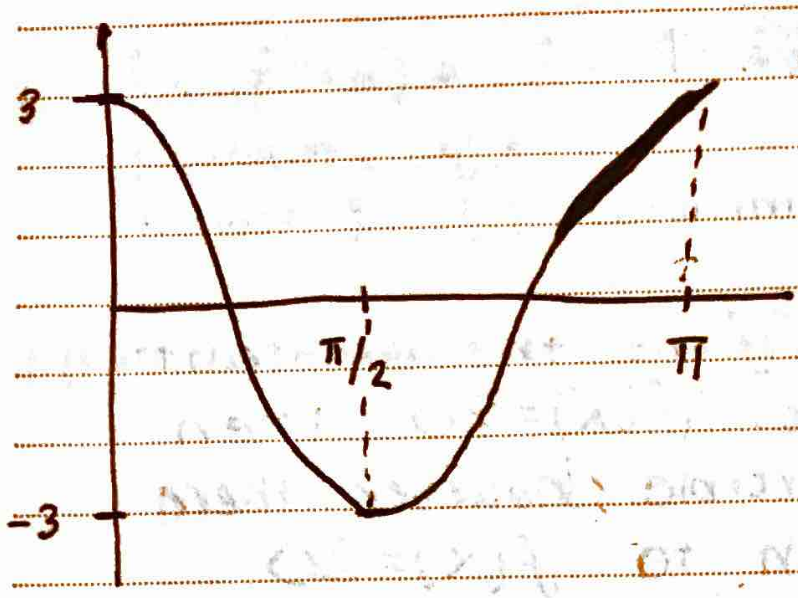
(13) False; let $f(x) = e^x$, $g(x) = x^2 + 1$ then $(f \circ g)(x) = e^{x^2+1}$ but $(g \circ f)(x) = (e^x)^2 + 1$

(14) True; by definition

(15) False; by problem #14

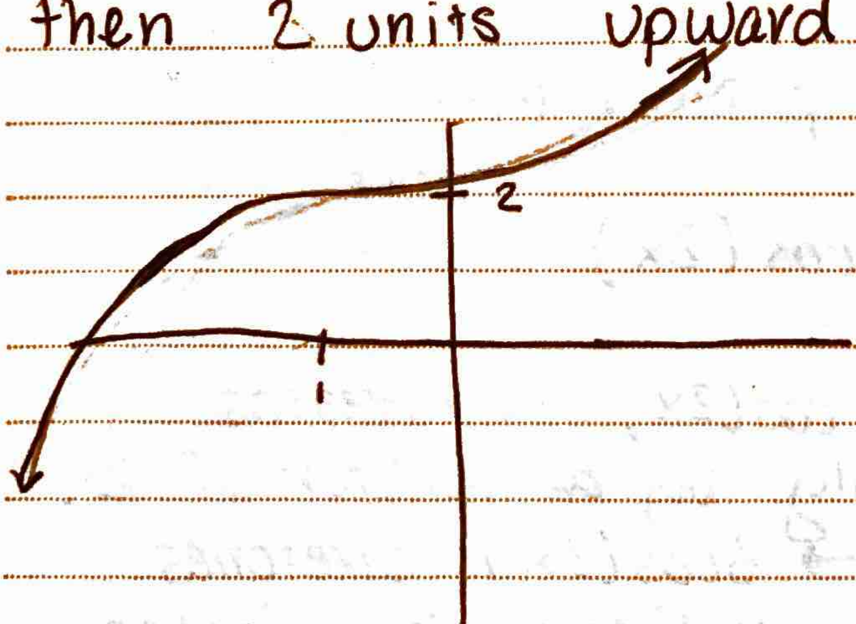
(16) (a) $f(x) = 3\cos(2x)$

Notice $\cos(x) \to \cos(2x)$ compresses $y = \cos x$ horizontally by a factor of 2. Then $\cos(2x) \to 3\cos(2x)$ stretches graph of $\cos(2x)$ vertically by factor of 3.



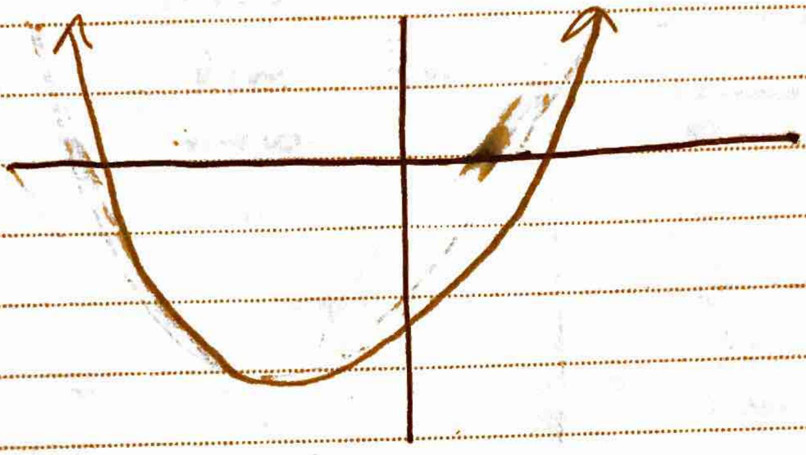
(b) $y = (x+1)^3 + 2$

We'll shift graph of x^3 1 unit to left then 2 units upward.



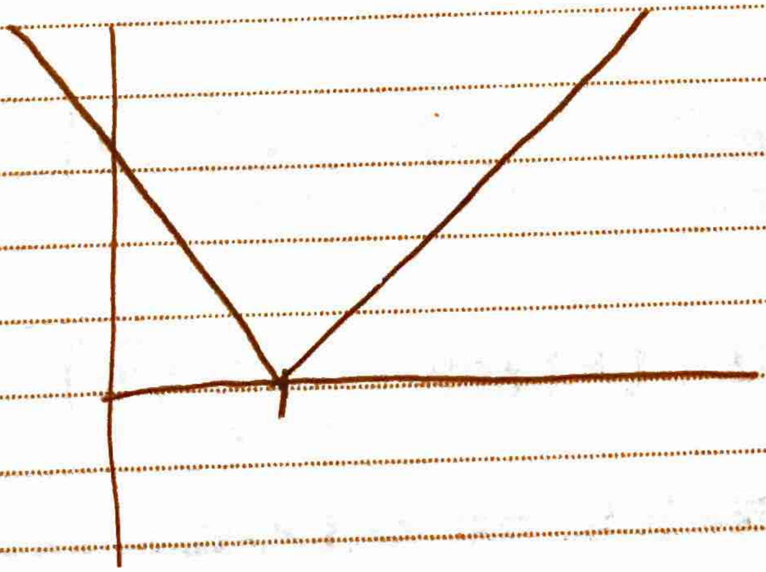
(c) $y = (2x+1)^2 - 3$

We'll compress graph of x^2 horizontally by factor of 2, shift it 1 unit to the left and 3 units down.



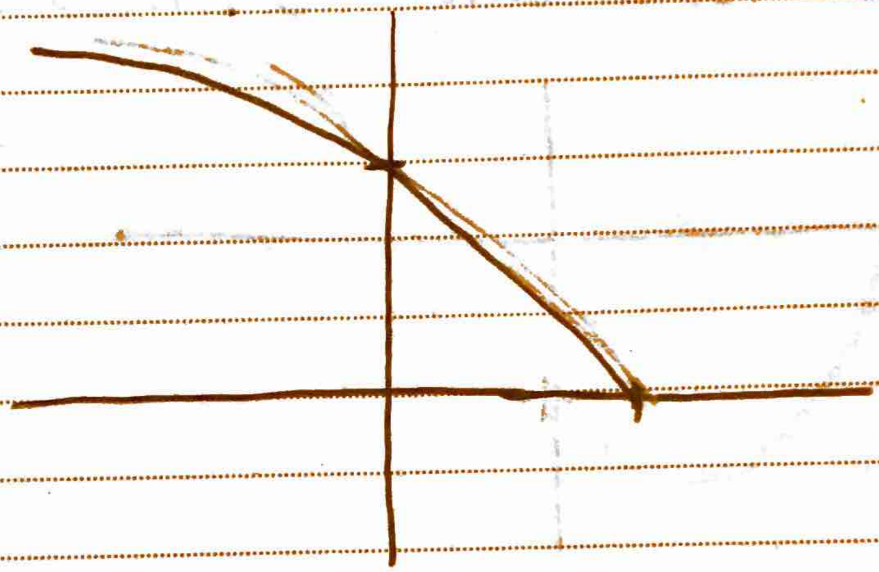
(d) $f(x) = |x-1|$

We'll shift graph of $y=|x|$ 1 unit to right



(e) $f(x) = \sqrt{1-x}$

We'll reflect $\sqrt{x}$ about y-axis to get $\sqrt{-x}$ then shift graph 1 unit to left.



(17) (a) $\lim\limits_{n\to\infty} \dfrac{n^2+1}{2n} = +\infty$

(b) $\lim\limits_{n\to\infty} e^n = +\infty$

(c) $\lim\limits_{n\to\infty} \dfrac{1/(5+n) - 1/5}{n}$

$= \lim\limits_{n\to\infty} \dfrac{5/5(5+n) - (5+n)/5(5+n)}{n}$

$= \lim\limits_{n\to\infty} \dfrac{5-(5+n)}{5n(5+n)}$

$= \lim\limits_{n\to\infty} \dfrac{-n}{25n+5n^2} = \lim\limits_{n\to\infty} \dfrac{-1}{25+5n} = 0$

$\cancel{= \lim\limits_{n\to\infty} \dfrac{-n}{n(30)} = \lim\limits_{n\to\infty} \dfrac{-1}{30} = \dfrac{-1}{30}}$

(d) $\lim\limits_{n\to\infty} \dfrac{2n^2+1}{4n^2-1} = \lim\limits_{n\to\infty} \dfrac{2+1/n^2}{4-1/n^2}$

$= \dfrac{2}{4} = \dfrac{1}{2}$

(e) $\lim\limits_{x\to\infty} \dfrac{|x|}{x^2} = \lim\limits_{x\to\infty} \dfrac{1}{x} = 0$

(f) $\lim\limits_{x\to\infty} \cos(x)$ DNE!

$\cos(x)$ is an oscillating function

(g) $\lim_{n\to\infty} \sqrt{n+1} - \sqrt{n} \cdot \frac{\sqrt{n+1} + \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$

$= \lim_{n\to\infty} \frac{(n+1) - n}{\sqrt{n+1} + \sqrt{n}}$

$= \lim_{n\to\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0$

(18) False; let $a_n = n$ and $b_n = -n$.

(19) (a) $\lim_{x \to 0} \frac{1}{x}$ DNE, notice that

if we approach 0 from left

we get $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ but

if we approach 0 from right

we see that $\lim_{x \to 0^+} \frac{1}{x} = +\infty$.

(b) $\lim_{x \to 1^+} \frac{1}{x^2-1} = +\infty$, notice that

there is vertical asymptote at $x=1$

& when we approach 1 from RHS

the function approaches $+\infty$

(c) $\lim_{x \to 0} \sin(x) = 0$, can see by

graphing $y = \sin(x)$

(d) $\lim_{x \to 2} \frac{\sqrt{x+2} - 2}{x-2} \cdot \frac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2}$

$= \frac{x+2-4}{(x-2)(\sqrt{x+2}+2)} = \frac{x-2}{x-2(\sqrt{x+2}+2)}$

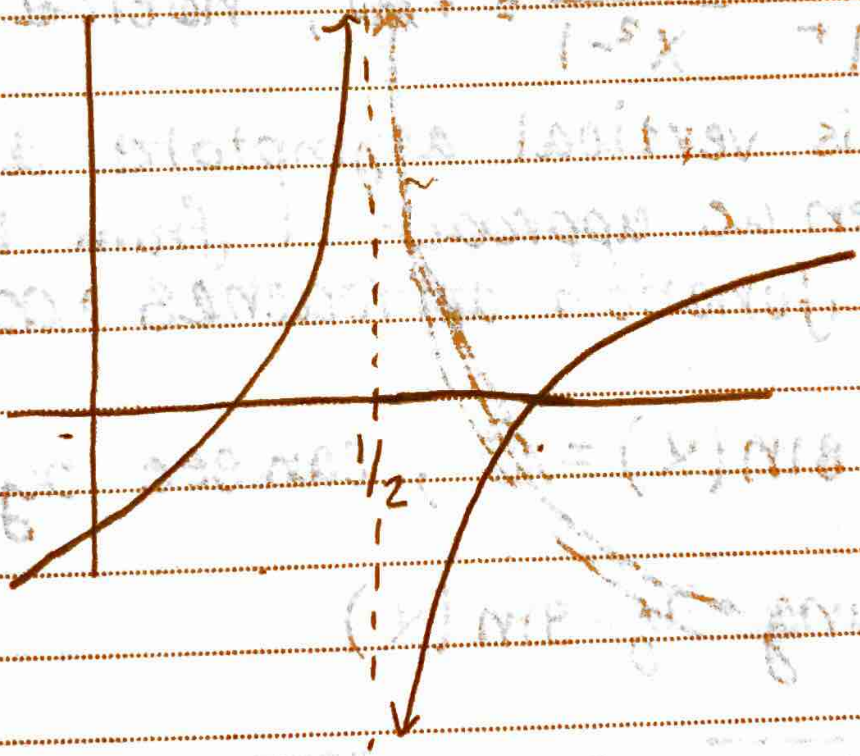
$\Rightarrow \lim_{x \to 2} \frac{1}{\sqrt{x+2}+2} = \frac{1}{4}$

(20) Yes. Notice that f is continuous everywhere except at $x=3$ (since f is a rational function).
Let $f(3)=6$ because:

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} (x+3) = 6.$$

Then f is continuous everywhere by construction.

(21)



(22) False, consider $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

(23) True by definition

(24) False, let $f(x) = \sin x$, $g(x) = \cos x$ on interval $(0, \pi)$. Clearly f & g are both continuous on $(0, \pi)$ but $f/g = \tan(x)$ is NOT!

(25) False, let $f(x) = (x-c)^2$ so that $f(c) = 0$. Clearly f is continuous at c.

(26) False, let $f(x) = \begin{cases} x^2 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

and $g(x) = 1$. Then $(g \circ f)(x) = 1$ is a continuous function.